UK Maths Trust

# Pink Kangaroo



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## Solutions

These are polished solutions and do not illustrate the process of failed ideas and rough work by which candidates may arrive at their own solutions.

It is not intended that these solutions should be thought of as the 'best' possible solutions and the ideas of readers may be equally meritorious.

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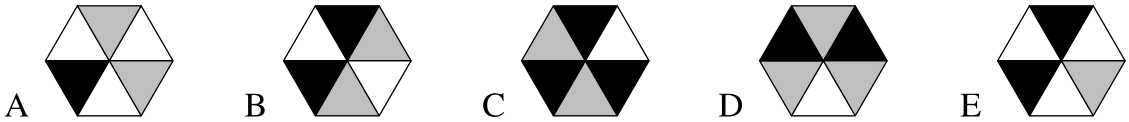
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| E | B | B | D | A | A | C | C | D | E | C | E | B | E | D | D | C | B | D | B | D | D | C | B | E |

**1.** Which of the following hexagons has exactly one-third of its area black and exactly half of its area white?




SOLUTION **E**

The hexagon is made up of six identical triangles. If one third of the hexagon is black that means that two of the triangles are black and if one half of the hexagon is white that means that three of the triangles are white.

Hexagon E is the only one that fulfils these two conditions.

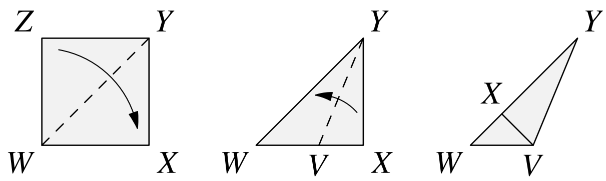
**2.** The base of a triangle increases by 50 % and its height decreases by one-third. What is the ratio of the area of the new triangle to that of the original triangle?

A 2 : 1   B 1 : 1   C 1 : 2   D 1 : 3   E 1 : 4

SOLUTION **B**

Let $b$ be the base of the original triangle and $h$ its height. Its area is equal to $\frac{bh}{2}$. The base of the new triangle is $\frac{3}{2}b$ and its height is $\frac{2}{3}h$. The new area is equal to $\frac{1}{2} \times \frac{3}{2}b \times \frac{2}{3}h$ which is $\frac{bh}{2}$. This is the same as the original area and so the ratio is 1 : 1.

**3.** Freddy folds a square $WXYZ$ in half along the diagonal $WY$ to make a triangle $WXY$. Then he folds the paper again so that the edge $XY$ of this triangle lies on part of the edge $WY$, making the smaller triangle $WVY$ as shown.




What is the size of angle $\angle WVY$?

A 108°   B 112.5°   C 120°   D 145°   E 157.5°

SOLUTION **B**

The right angle at the corner $Y$ is bisected by the first fold, so $\angle WYX = 45°$ and also $\angle YWX = 45°$. Similarly $\angle WYX$ is bisected by the second fold and so $\angle WYV = \frac{45}{2}°$, which is 22.5°.

Considering the angles in triangle $YWV$ shows that $\angle WVY = 180° - 45° - 22.5° = 112.5°$.

**4.** The four-digit number 80□□ is missing its last two digits. It is divisible by 8 and also by 9. What is the product of its last two digits?

A 6 B 16 C 20 D 24 E 48

SOLUTION **D**

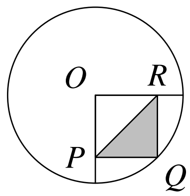
Since 8000 is divisible by 8 the last two digits must form a number divisible by 8.

Since the number 80□□ is divisible by 9, the sum of its digits is a multiple of 9. There are only two possible ways this could happen, namely $8 + 0 + \square + \square = 9$ or $8 + 0 + \square + \square = 18$.

So either $\square + \square = 1$ or $\square + \square = 10$. Checking the possible pairs of numbers which also form a 2-digit multiple of 8, we find that only 64 works. So the only 4-digit number divisible by both 8 and 9 starting with 80 is 8064. Hence the product of the two last digits is 24.

**5.** A circle with centre $O$ and radius 10 cm is given. A square $OPQR$ is drawn inside the circle, where $Q$ is a point on the circle.

What is the area of the shaded triangle $PQR$?




A $25\,\text{cm}^2$ B $30\,\text{cm}^2$ C $35\,\text{cm}^2$ D $40\,\text{cm}^2$ E $50\,\text{cm}^2$

SOLUTION **A**

We know that $OQ$ is 10 cm as it's a radius of the circle. Consider the right-angled triangle $PRQ$. Since $PR$ and $OQ$ are diagonals of the square, $PR$ also has length 10 cm. By Pythagoras, $PR^2 = PQ^2 + QR^2$. Also $PQ = QR$ since they are sides of the square. Therefore $2PQ^2 = PR^2 = 100\,\text{cm}^2$ and so $PQ^2 = 50\,\text{cm}^2$. Therefore the area of triangle $PQR = \frac{1}{2} \times PQ \times QR = \frac{1}{2}PQ^2 = 25\,\text{cm}^2$.

**6.** Sam has some dogs, some rabbits and some cats. Eight of his pets are not dogs, five of them are not rabbits, and seven of them are not cats.

How many pets does Sam have?

A 10 B 11 C 15 D 16 E 20

SOLUTION **A**

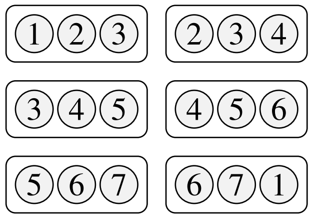
One of Sam's rabbits will be counted among the 8 which are not dogs and the 7 which are not cats. A similar comment applies for any cat or dog. So the sum $8 + 5 + 7 = 20$ counts each pet twice. So Sam has 10 pets.

ALTERNATIVE

We take an algebraic approach. Let $r$ be the number of rabbits among his pets, $d$ be the number of dogs and $c$ be the number of cats.

Then $r + c = 8$, $d + c = 5$ and $d + r = 7$. Adding these equations, we get $2(r + c + d) = 20$. Hence $r + c + d = 10$.

**7.** Aimee has a collection of two gold and five silver medals. They are numbered from 1 to 7, in some order. The diagram shows six black and white photos of the medals. It is known that in each photo, exactly one of the medals is gold. What is the sum of the numbers on the two gold medals?




A 7 B 8 C 9 D 10 E 11

SOLUTION **C**

A look at the photos of the medals reveals that each of 3, 4, 5 and 6 appears three times and each of 1, 2 and 7 appears twice. Overall gold appears six times, once on each photo. Therefore each gold medal must appear three times. Hence 1, 2 and 7 must be silver. Since 1, 2 and 3 are on one photo, 3 must be gold and since 6, 7 and 1 are on another photo, 6 must be gold. The gold medal numbers 3 and 6 have sum 9, so option C is correct.

**8.** Sami and Parnika are celebrating their birthday today. Sami notices that $\frac{1}{19}$ of Parnika's age is equal to $\frac{1}{17}$ of his age. The sum of their ages is greater than 40 and less than 100. How old is Parnika?

A 19 B 34 C 38 D 57 E 76

SOLUTION **C**

Let Sami's age be $s$ and Parnika's age be $p$. Hence $\frac{1}{19}s = \frac{1}{17}p$, and $17s = 19p$. Since 17 and 19 are prime, 17 must divide $p$ and 19 must divide $s$.Therefore $s = 19 \times k$ and $p = 17 \times k$, for some integer $k$. From the question we know that

$$40 < 17k + 19k < 100,$$

that is

$$40 < 36k < 100,$$

that is

$$\frac{40}{36} < k < \frac{100}{36},$$

that is

$$1\frac{4}{36} < k < 2\frac{28}{36}.$$

The only integer that satisfies these inequalities is $k = 2$, so Sami is $19 \times 2$, which is 38.

**9.** Debanshi has a bag of 18 balls, numbered from 1 to 18. What is the smallest number of balls she would have to remove to be sure that at least three of them displayed primes?

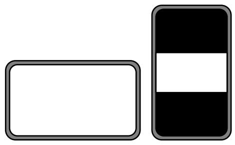
A 11 B 12 C 13 D 14 E 15

SOLUTION **D**

There are seven primes less than 18, namely 2, 3, 5, 7, 11, 13 and 17. Therefore there are eleven composite numbers.

If she takes at least fourteen balls then, even if they include all the composite numbers, she must have taken three primes. But if she chooses just thirteen balls then she might have all the composite numbers plus two primes. This means she has to take 14 balls. So D is true.

**10.** Etienne looks at a photo on his smartphone. The width and height of the photo are in the ratio of 16 : 9 and fills the whole display. When he holds the smartphone vertically, the photo is reduced in size. What proportion of the display area is taken up by the smaller photo?



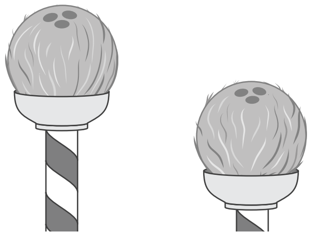
A $\frac{3}{4}$ B $\frac{9}{16}$ C $\frac{27}{64}$ D $\frac{32}{81}$ E $\frac{81}{256}$

Solution **E**

The width of the photo in the left picture has the same length as the long side of the display. And the width of the photo in the right picture is as long as the short side of the display. Therefore, the side-lengths of the photo in the right picture are $\frac{9}{16}$ of the side-lengths of the photo in the left picture. Hence, the photo on the right takes up $\left(\frac{9}{16}\right)^2$ of the whole display, that is $\frac{81}{256}$.

**11.** Shruti throws 27 balls, each time at one or other of the two coconuts. She hits with 50 % of her throws aimed at the top left coconut and 80 % of her throws at the bottom right coconut. Nine of her throws miss their target.

How many times does she hit the top left coconut?



A 4 B 5 C 6 D 7 E 8

Solution **C**

Let $a$ be the number of times Shruti throws at the top left coconut and $b$ the number of times she throws at the bottom right coconut.

She throws either at the top left or bottom right coconut, so $a + b = 27$. By considering the number of times she misses, we get $\frac{1}{2}a + \frac{1}{5}b = 9$.

We have

$$a + b = 27 \qquad (1)$$

and

$$\frac{1}{2}a + \frac{1}{5}b = 9 \qquad (2)$$

Subtracting equation (1) from 5 × equation (2) gives $\frac{3}{2}a = 45 - 27 = 18$. That gives $a = 12$.

Hence the number of times she hits the top coconut is $\frac{1}{2} \times 12 = 6$.

**12.** The integer $N$ is the largest six-digit integer with the product of all its digits equal to 180. What is the sum of the digits of $N$?

A 16 B 17 C 18 D 20 E 21

SOLUTION **E**

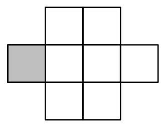
Since we want the largest number and 9 divides 180, the first digit of $N$ is 9.

The product of the next five digits is $180 \div 9$ which is 20. The largest single digit factor of 20 is 5 so the second digit of $N$ is 5.

This leaves the last four digits to have a product of 4. The largest divisor is 4 meaning the last four digits must be 4, 1, 1 and 1.

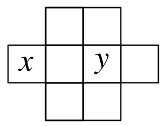
The number therefore is 954 111, which has a digit sum of $9 + 5 + 4 + 1 + 1 + 1$ which is 21.

**13.** Prem wants to place the digits 1 to 8, inclusive, in the eight cells of the diagram. He wants to place one digit in each cell so that any two cells that contain consecutive digits are not adjacent (including diagonally). What digits could be placed in the shaded cell?



A 1 or 8 B 2 or 7 C 3 or 6 D 4 or 5 E 7 or 8

SOLUTION **B**




Each of the two central cells is adjacent to all but one of the other cells. Except for 1 and 8, each digit has two other digits to which it is consecutive. So 1 and 8 must be placed in the central cells. If 1 is placed in the cell labelled $y$, then 2 must be in cell $x$. Similarly, if 8 is placed in $y$ then 7 must be in $x$.

Therefore the digits that could be placed in the shaded cell are 2 and 7. This excludes all options except B. It is left to the reader to check that, for each of 2 and 7, the diagram can be completed with that digit in the shaded cell.

**14.** The product of three primes is 11 times as big as their sum. What is the largest possible value that this sum could take?

A 14 B 17 C 21 D 25 E 26

SOLUTION **E**

Let $p$, $q$ and $r$ be our primes. We are looking to satisfy the equation $pqr = 11(p + q + r)$. Because 11 is prime, the only way for $pqr$ to be a multiple of 11 is if one of $p$, $q$ and $r$ is 11. Without loss of generality we can assume $r$ is 11. Substituting this into the above equation:

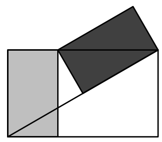
$$11pq = 11(p + q + 11);$$

that is

$$pq = p + q + 11.$$

If $p$ and $q$ are both greater than or equal to 5, then the left-hand side of the equation is greater than the right-hand side. So one of $p$ and $q$ must be either 2 or 3. Therefore the possible values of $(p, q, r)$ are $(2, 13, 11)$ and $(3, 7, 11)$. Hence the maximal value of $S$ is $2 + 11 + 13$, which is 26.

**15.** The two shaded rectangles are identical. They both have an area of $4\,\text{cm}^2$.



What is the area of the large rectangle?

A $10\,\text{cm}^2$ B $8\sqrt{3}\,\text{cm}^2$ C $8\,\text{cm}^2$ D $12\,\text{cm}^2$ E $4\sqrt{3}\,\text{cm}^2$

SOLUTION **D**

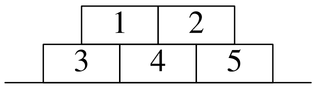
The two small triangles in the lower left of the diagram have three angles and a side (the short side of the rectangles) in common. Therefore they are congruent and so have equal areas.

Now consider the large triangle which is the top half of the large rectangle. Because the two small triangles have equal areas, we see that the area of the large triangle equals the area of the light rectangle plus half the area of the dark grey rectangle. Hence the area of the large rectangle is equal to three times the area of one of the small rectangles.

So its area is $4 \times 3\,\text{cm}^2$, which is $12\,\text{cm}^2$.

**16.** Five bricks are placed on the ground, as shown on the right. Navithan can only remove a brick if there are no bricks on top of it. He repeatedly removes a brick, chosen at random from those available, until all the bricks are removed.




What is the probability that the brick numbered 4 is the third one to be removed?

A $\frac{1}{3}$ B $\frac{1}{4}$ C $\frac{1}{5}$ D $\frac{1}{6}$ E $\frac{1}{8}$

SOLUTION **D**

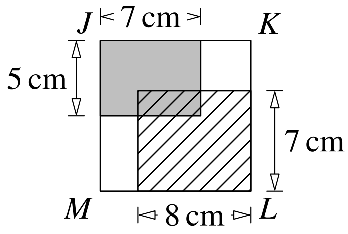
To remove the brick labelled with 4, Navithan needs to remove the top row of bricks. In order to remove block 4, he has to remove 1 and 2 in any order, then remove 4.

By symmetry, it doesn't matter which brick Navithan removes first, so let's assume he removes brick 1. He can choose between 2 and 3 for the next brick, choosing 2 with a probability $\frac{1}{2}$.

To then pick brick 4, he must choose from the three remaining with probability $\frac{1}{3}$.

So the probability that the brick numbered 4 is the third one to be removed is $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$.

**17.** Square $JKLM$ contains two rectangles. One is grey and the other striped, with dimensions as shown, but not to scale. The area where the rectangles overlap is $18\,\text{cm}^2$. What is the perimeter of $JKLM$?




A 28 cm B 34 cm C 36 cm D 38 cm E 40 cm

Solution **C**

Let the side of the given square be $x$ cm. Let the height of the area of overlap be $h$ cm. Then $x = 5 - h + 7$, so $h = 12 - x$. Similarly, we can show that the length of the overlapping part is $(15 - x)$ cm.

The area where the rectangles overlap is 18 cm. Equating this with our width and height found previously,

$$(15 - x)(12 - x) = 18;$$

that is

$$180 - 27x + x^2 = 18;$$

that is

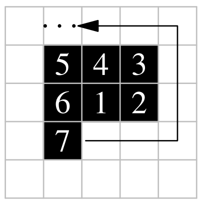
$$x^2 - 27x + 162 = 0;$$

that is

$$(x - 9)(x - 18) = 0.$$

Hence, $x = 9$ or $x = 18$. If $x = 18$, then the two rectangles would not overlap. So, $x = 9$.Therefore the perimeter of the square is $4 \times 9 = 36$ cm.

**18.** Patrick numbers certain squares on a sheet of grid paper. Each square has a side-length of 0.5 cm. He starts by putting 1 in one of the squares and then numbers $2, 3, 4, 5, \ldots$ in squares in an anti-clockwise direction, as shown. He stops when he has numbered 2025 squares, and looks at the figure made up of all numbered squares. What is the perimeter of this figure?

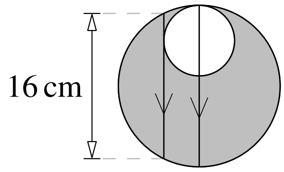



A 80 cm B 90 cm C 120 cm D 180 cm E 360 cm

Solution **B**

When Patrick reaches a square number the figure formed will be a square. As 2025 is equal to $45^2$, Patrick will stop when his figure is a square with sides of length $45 \times 0.5$ cm = 22.5 cm. Hence the perimeter, in cm, is $22.5 \times 4$, which is 90 cm.

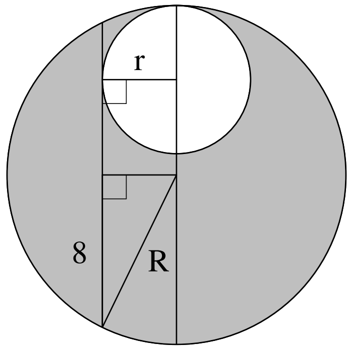
**19.** The figure shows a diameter of a smaller circle which lies along the diameter of a larger touching circle. It also shows a chord of the larger circle which is parallel to these diameters and is tangent to the smaller circle. The length of the chord is 16 cm. What is the area of the shaded region?




A $16\pi$ cm$^2$ B $36\pi$ cm$^2$ C $49\pi$ cm$^2$ D $64\pi$ cm$^2$ E $81\pi$ cm$^2$

Solution **D**

Let the radius of the outer circle be $R$ cm and the radius of the inner circle be $r$ cm. So the shaded region has area $\pi(R^2 - r^2)$ cm$^2$.




The two parallel lines are $r$ cm apart. We can therefore construct a right-angled triangle, as shown in the diagram. By Pythagoras' theorem, $R^2 - r^2 = 8^2$, so the shaded region has area $64\pi$ cm$^2$.

**20.** Eghosa has written down a six-digit number '$abcdef$' where the six digits are 1, 2, 3, 4, 5, and 6 in some order. The first two digits form a number '$ab$' which is a multiple of 2. The first three digits form a number '$abc$' which is a multiple of 3. Similarly, '$abcd$' is a multiple of 4, '$abcde$' is a multiple of 5, and '$abcdef$' is a multiple of 6.

What are the possible values for the sixth digit $f$?

A 2 B 4 C 6 D 2 or 4 E 4 or 6

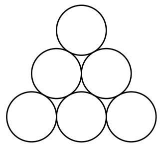
Solution **B**

For '$abcde$' to be a multiple of 5, $e$ must be 5. To make sure that '$ab$', '$abcd$' and '$abcdef$' are even, the digits $b$, $d$ and $f$ must be come from $\{2, 4, 6\}$. Hence, $a$ and $c$ must be the remaining digits $\{1, 3\}$.

This means that '$abc$' has to consist of 1, 3 and $b$ which is one of 2, 4 or 6. For '$abc$' to be a multiple of three, the sum $a + b + c$ must equal a multiple of 3 and only $b = 2$ satisfies this condition.

For '$abcd$' to be a multiple of four, the last two digits, '$cd$', must be a multiple of four. The only possible combinations we have are $\{14, 16, 34, 36\}$, of which only $\{16, 36\}$ give a multiple of 4. Hence, $d = 6$ and this means that $f = 4$.

**21.** Six circles are arranged in the shape of a triangle. Louis is asked to write the positive integers from one to six, inclusive, in the circles so that the sums of the numbers on each side of this triangle are the same. He is then asked to calculate the sum of the three integers at the vertices of the triangle. How many different sums could Louis calculate?



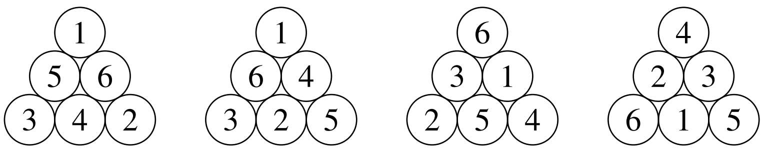
A 1 B 2 C 3 D 4 E 5

Solution **D**

The sum of the six inscribed numbers is $1 + 2 + 3 + 4 + 5 + 6 = 21$.

The sum of the numbers at the vertices of the triangle is not less than $1 + 2 + 3 = 6$, but not more than $4 + 5 + 6 = 15$.

The numbers at the vertices are included twice in the totals of the numbers on the sides of the triangle. Let their sum be $T$. Then the three totals of the numbers on the sides of the triangle sum to $21 + T$. As these three totals are equal, we can deduce that $T$ is a multiple of 3. It has been shown already that $6 \leq T \leq 15$, so the only possible values of $T$ are 6, 9, 12 and 15. The following examples show that all four of these values are possible.



**22.** Ella is constructing a sequence of numbers such that, starting from the third term, each term is equal to the average of all the previous terms. That is, the third term is the average of the first and second terms; the fourth term is the average of the first, second and third terms; and so on. She starts her sequence with 8 and the tenth term she writes down is 26. What is the second term?

A 28 B 32 C 38 D 44 E 50

SOLUTION **D**

Let the second term of the sequence be $a$. The average of the first two is $\frac{8+a}{2}$, and so the first three terms are

$$8,\ a,\ \frac{8+a}{2}.$$

The sum of these three terms is,

$$8+a+\frac{8+a}{2}=\frac{3(8+a)}{2}.$$

This has the same sum as if the sequence was instead made up of the following three terms:

$$\frac{8+a}{2},\ \frac{8+a}{2},\ \frac{8+a}{2}$$

and it follows that the average of the first three terms is also $\frac{8+a}{2}$. So the first four terms of the sequence are,

$$8,\ a,\ \frac{8+a}{2},\ \frac{8+a}{2}.$$

These have the same sum as the four terms:

$$\frac{8+a}{2},\ \frac{8+a}{2},\ \frac{8+a}{2},\ \frac{8+a}{2}.$$

It follows that the average of the first four terms is also $\frac{8+a}{2}$ and therefore this is the fifth term of the sequence, and so on. Thus after the first two terms each term of the sequence is equal to the mean of the first two terms.

In this sequence we know that the tenth term is 26. It follows that the 26 is the mean of the first two terms. Hence their sum is $2 \times 26 = 52$. Since the first term is 8, we deduce that the second term is $52 - 8$ which is 44.

**23.** At a party, there are twelve children, among whom there are three pairs of twins. Shreya has six blue hats and six red hats. In how many different ways can she choose which colour of hat to give to each child while ensuring that, for each pair of twins, the two are given the same colour?

A 72 B 86 C 92 D 102 E 132

SOLUTION **C**

Let us assume the majority of the twins are wearing red hats.

**Case 1:** *All the twins are wearing the red hats.* There is one way of doing this.

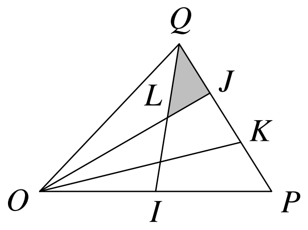
**Case 2:** *Exactly two pairs of twins are wearing red hats.* Each pair of twins could be the ones wearing the blue hats, so the two pairs of twins can be chosen in three ways.

This leaves two red hats to be distributed amongst the remaining 6 children. There are six choices for the first hat and five for the second. We have to divide by two because the order doesn't matter. So there are $\frac{1}{2} \times 6 \times 5 = 15$ ways to distribute the remaining two hats.

Hence there are $3 \times 15 = 45$ ways of exactly two pairs of twins wearing red hats

Hence there are $45 + 1 = 46$ ways in which Shreya can give the majority of the twins red hats. Similarly there are 46 ways in which she can give the majority of the twins blue hats. So, in total, there are $46 + 46 = 92$ ways in which Shreya can choose the hats.

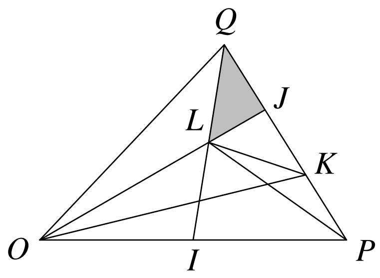
**24.** Triangle $OQP$ has an area of $60\,\text{cm}^2$. $I$ is the midpoint of side $OP$, and the points $J$ and $K$ divide side $QP$ into three equal segments. Let $L$ be the intersection of $QI$ and $OJ$. What is the area of triangle $QLJ$?




A $4\,\text{cm}^2$ B $5\,\text{cm}^2$ C $6\,\text{cm}^2$ D $7\,\text{cm}^2$ E $8\,\text{cm}^2$

Solution **B**

It is useful to draw in edges $LP$ and $LK$ to the diagram.




Triangles $QJL$, $JLK$ and $KLP$ have the same area as they share a common base length and height from shared point $L$. Let's denote this as $x\,\text{cm}^2$.

Similarly $OLI$ and $LIP$ also have equal areas. Let's call this $y\,\text{cm}^2$.

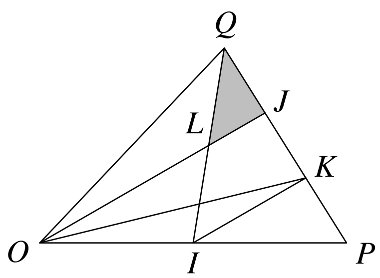
The area of the triangle $QIP$ is half of the area of the triangle $OQP$, so it has an area of $30\,\text{cm}^2$. This triangle can be composed from four of the triangles we found earlier. That is $QJL$, $JLK$, $KLP$ and $LIP$. We can therefore formulate the expression $3x + y = 30$.

Similarly the triangle $JOP$ has an area of two-thirds of the triangle $OQP$. Again we can formulate an expression from the triangles $OLI$, $LIP$, $JLK$ and $KLP$ as $2x + 2y = 40$.

Solving these equations simultaneously gives $x = 5$ and $y = 15$. Hence the area of the shaded region is $5\,\text{cm}^2$ which is B.

Alternative

We could instead draw in the line $IK$.The area of the triangle $QIP$ is half the area of the triangle $OQP$, so it has an area of $30\,\text{cm}^2$. Similarly, $OKP$ is a third of the area of the triangle $OQP$, so it has an area of $20\,\text{cm}^2$.




Triangle $IKP$ is half the area of the triangle $OKP$, so it has an area of $10\,\text{cm}^2$. It is possible to calculate the area of the triangle $QIK$ by subtracting the areas of triangles $QIP$ and $IKP$. This gives the area as $(30 - 20)\,\text{cm}^2 = 10\,\text{cm}^2$.

We can now make use of the midpoint theorem, which states that the line joining the midpoints of two sides of a triangle is parallel to the third side. Because $K$ and $I$ are the midpoints of sides $JP$ and $OP$ respectively, we can conclude that $IK$ is parallel to $OJ$.

This means that triangles $QLJ$ and $QIK$ are similar. Side $QJ$ is half of side $QK$, which gives a linear scale factor of $\frac{1}{2}$. Hence, the area scale factor is $\left(\frac{1}{2}\right)^2$ which is $\frac{1}{4}$. Hence the area of $QLJ$ is $\frac{1}{4} \times$ area $QIK$. This is $\frac{1}{4} \times 20$ which is 5.

**25.** Chiakala wants to write the positive integers from one to eight, inclusive, into the squares of a $2 \times 4$ grid. For each square, if there is a neighbouring square to its right or below, that neighbour must contain a number larger than that in the given square. In how many different ways can Chiakala fill the grid?

| | | | |
|---|---|---|---|
| | | | |
| | | | |

A 6 B 8 C 10 D 12 E 14

SOLUTION **E**

The number 1 has to be placed in the top left cell as there are no smaller numbers available to go to the right or above it. Similarly the 8 must be in the bottom right square as it is the biggest of all the numbers to be placed. The numbers 2 and 7 the must be adjacent to the 1 and 8 respectively. This gives four possible arrangements, as shown in the four cases below:

| | | | |
|---|---|---|---|
| 1 | 2 | | |
| | | 7 | 8 |

| | | | |
|---|---|---|---|
| 1 | 2 | | 7 |
| | | | 8 |

| | | | |
|---|---|---|---|
| 1 | | | |
| 2 | | 7 | 8 |

| | | | |
|---|---|---|---|
| 1 | | | 7 |
| 2 | | | 8 |

We then have to place the other digits. Let's consider each case in turn.

**Case 1** In each row, the two missing numbers can be any of the two remaining four numbers as long as the order is consistent. The cell labelled $a$ must be one of $\{3, 4, 5\}$. If $a$ is 3, $b$ must be one of $\{4, 5, 6\}$. If $a$ is 4, $b$ must be from $\{5, 6\}$. If $a$ is 5, $b$ must be 6. There are six options here.

| | | | |
|---|---|---|---|
| 1 | 2 | $a$ | $b$ |
| | | 7 | 8 |

**Case 2** Here the 6 must be in the cell adjacent to 8 so has to be placed in the second row, third column. Any of the three numbers from $\{3, 4, 5\}$ can fill in cell labelled $a$, and the other two must then be placed on the second row in ascending order. There are three options here.

| | | | |
|---|---|---|---|
| 1 | 2 | $a$ | 7 |
| | | 6 | 8 |

**Case 3** Here the 3 must be placed in the first row, second column. As in **Case 2**, the cell labelled $a$ must contain one of $\{4, 5, 6\}$ and the other two must be placed in the first row in ascending order. There are three options here.

| | | | |
|---|---|---|---|
| 1 | 3 | | |
| 2 | $a$ | 7 | 8 |

**Case 4** Here both the 3 and the 6 be placed as shown. The 4 and the 5 can then be placed in either of the empty cells giving two more options.

| | | | |
|---|---|---|---|
| 1 | 3 | | 7 |
| 2 | | 6 | 8 |

In total there are $6 + 3 + 3 + 2 = 14$ possible arrangements, which is E.